# Stability Analyses in Geotechnical Engineering (by Dr J. Takemura) 

## Mid-term Exam: 17 ${ }^{\text {th }}$ November 2003

Question 1\& 2: Due on 10:30 17 ${ }^{\text {th }}$
Question 3: Due on 10:30 18 ${ }^{\text {th }}$

1. Obtain the active total earth pressure $P_{a}$ on a vertical smooth surface wall by answering the following questions. The soil behind the wall has unit weight $\gamma_{\mathrm{t}}$, cohesion $\mathrm{c}^{\prime}$ and friction angle $\phi^{\prime}(=$ dilation angle $\psi)$ and the wall height is $H$. Assume plane strain (twodimensional) condition in all solutions.
(1) Obtain the permissible stress field for this problem by showing the Mohr stress circle of the element $A$ at the depth z. (Fig.1)
(2) Obtain the lower bound of the active total earth pressure, $P_{a l}$.
(3) Draw the displacement diagram of the straight surface failure mechanism shown in Fig. 2 using the notations of the horizontal displacement of wall, $\delta h$, and the optimization parameter, $\alpha$.
(4) Obtain the external work $\Delta E$ as a function of $P_{a u}, \gamma_{t}, \phi^{\prime}, H, \delta h$ and $\alpha$.
(5) Obtain the internal dissipation $\Delta W$ as a function of $P_{a u}, \mathbf{c}^{\prime}, \phi^{\prime}, \mathrm{H}, \delta \mathrm{h}$ and $\alpha$.
(6) Obtain the upper bound of the active total earth pressure $\boldsymbol{P}_{a u}$ as a function of $\boldsymbol{\gamma}_{\mathrm{t}}, \mathrm{c}^{\prime}$, $\phi^{\prime}, H$ and $\alpha$.
(7) Obtain $\alpha$ satisfying $d P_{a v} / d \alpha=0$ and confirm that the upper bound of $P_{a u}$ with the obtained $\alpha$ is equal to the lower bound of active pressure, $P_{a l}$ and these are exact solution of this problem.


Fig. 1


Fig. 2
2. Explain the reasons why limit analysis can be reasonably applied for stability analysis on clay in short term problems and cannot be directly applied for that on loose sand.
3. Consider the bearing capacity of rigid strip (2D) footing on the top edge of clay ( $c_{u}, \phi_{u}=0$, $\gamma$ ) slope, which is underlain by rock as shown in the Fig.2. In order to evaluate the bearing capacity by upper bound analysis, two failure mechanisms (Mechanism $I$ and Mechanism II) are used as shown in Fig.3. Height of the slope is $\sqrt{3} B / 2$, where B is width of the footing, and the slope angle is $60^{\circ}$. Answer the following questions.
(1) Draw the displacement diagrams for the two mechanisms. Use $\delta w_{F}$ as the notation for the vertical component of displacement of the footing in the displacement diagram.
(2) Obtain the external works done by boundary forces and self-weight of the soil for the two mechanisms using given parameters in Fig.3.
(3) Obtain the internal energy dissipations for the two mechanisms using given parameters in Fig.3.
(4) Obtain the bearing capacities $\left(Q_{\text {utt }}\right)$ for the two mechanisms.

Fig. 4 shows the slip line network on this problem.
(5) What are the type of the slip line $g f$ ( $\alpha$ line or $\beta$ line) and the reason of the answer?
(6) Draw the Mohr stress circles with the locations the pole at the points $a, h$ and $c$ along the slip line ac.
(7) Obtain the ultimate bearing pressure distribution at the footing base $q_{u l t}(x)$ and compare the bearing capacity $Q_{\text {ult }}$ with the upper bound of bearing capacity obtained from Mechanism II.


Mechanism I


Mechanism II

Fig. 3


Fig. 4

